

MSC: 47A10, 47B37.

**ON THE SPECTRUM OF THE OPERATOR Δ_v OVER THE SPACE l_p ,
($1 < p < \infty$)**

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The main purpose of this paper is to determine the spectrum of the generalized difference operator Δ_v over the sequence space l_p , where $1 < p < \infty$. The operator Δ_v on the space l_p is defined by $\Delta_v x = \Delta_v(x_k) = (v_k x_k - v_{k-1} x_{k-1})_{k=0}^{\infty}$ with $x_{-1} = v_{-1} = 0$, where $x = (x_k) \in l_p$ and (v_k) is a strictly decreasing sequence of positive real numbers satisfying certain conditions. In this paper, some results concerning the spectrum, the point spectrum, the residual spectrum and the continuous spectrum of the operator Δ_v on the space l_p have been found.

Key words: spectrum of an operator, Generalized difference operator, The sequence space l_p .

1. Preliminaries, background and notation

Let X and Y be Banach spaces and $T : X \rightarrow Y$ be a bounded linear operator. By $R(T)$, we denote the range of T , i.e,

$$R(T) = \{y \in Y : y = Tx, x \in X\}.$$

By $B(X)$, we denote the set of all bounded linear operators on X into itself. If $T \in B(X)$, then the adjoint T^* of T is a bounded linear operator on the dual X^* of X defined by $(T^*f)(x) = f(Tx)$, for all $f \in X^*$ and $x \in X$.

We shall need some basic concepts in spectral theory which are given as follows (see [11, pp. 370-371]).

Let $X \neq \{\emptyset\}$ be a complex normed space and $T : D(T) \rightarrow X$ be a linear operator with domain $D(T) \subseteq X$. With T we associate the operator

$$T_\lambda = T - \lambda I,$$

where λ is a complex number and I is the identity operator on $D(T)$. If T_λ has an inverse, which is linear, we denote it by T_λ^{-1} , that is

$$T_\lambda^{-1} = (T - \lambda I)^{-1},$$

and call it the *resolvent operator* of T . A *regular value* λ of T is a complex number such that

(R1) T_λ^{-1} exists,

(R2) T_λ^{-1} is bounded,

(R3) T_λ^{-1} is defined on a set which is dense in X .

The *resolvent set* of T , denoted by $\rho(T, X)$, is the set of all regular values λ of T . Its complement $\sigma(T, X) = C \setminus \rho(T, X)$ in the complex plane C is called the *spectrum* of T . Furthermore, the spectrum $\sigma(T, X)$ is partitioned into three disjoint sets as follows:

The *point (discrete) spectrum* $\sigma_p(T, X)$ is the set such that T_λ^{-1} does not exist. Any such $\lambda \in \sigma_p(T, X)$ is called an *eigenvalue* of T .

The *continuous spectrum* $\sigma_c(T, X)$ is the set such that T_λ^{-1} exists and satisfies (R3) but not (R2), that is, T_λ^{-1} is unbounded.

The *residual spectrum* $\sigma_r(T, X)$ is the set such that T_λ^{-1} exists (and may be bounded or not) but does not satisfy (R3), that is, the domain of T_λ^{-1} is not dense in X .

By w , we shall denote the space of all real or complex valued sequences. Any vector subspace of w is called a *sequence space*. We shall write l_∞, c, c_0 and bv for the spaces of all bounded, convergent, null and bounded variation sequences, respectively. Also by l_1, l_p and bv_p we denote the spaces of all absolutely, p -absolutely summable sequences and p -bounded variation sequences, respectively.

Let λ and μ be two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk} , where $n, k \in N = \{0, 1, 2, \dots\}$. Then, we say that A defines a matrix mapping from λ into μ , and we denote it by $A: \lambda \rightarrow \mu$, if for every sequence $x = (x_k) \in \lambda$, the sequence $Ax = \{(Ax)_n\}$, the A -transform of x , is in μ , where

$$(Ax)_n = \sum_k a_{nk} x_k, \quad (n \in N). \quad (1.1)$$

For simplicity in notation, here and in what follows, the summation without limits runs from 0 to ∞ . By (λ, μ) , we denote the class of all matrices A such that $A: \lambda \rightarrow \mu$. Thus, $A \in (\lambda, \mu)$ if and only if the series on the right side of (1.1) converges for each $n \in N$ and every $x \in \lambda$, and we have $Ax = \{(Ax)_n\}_{n \in N} \in \mu$ for all $x \in \lambda$. We assume throughout unless stated otherwise that $p, q > 1$ with $p^{-1} + q^{-1} = 1$ and we use the convention that any term with negative subscript is equal to naught.

The fine spectrum of the difference operator Δ over the sequence spaces c_0 and c has been determined by Altay and Başar [5]. Akhmedov

and Başar [1, 2] have studied the fine spectrum of the difference operator Δ over the sequence spaces l_p and bv_p , where $1 \leq p < \infty$. Note that the sequence space bv_p was introduced and studied by Başar and Altay [8] and Akhmedov and Başar [2]. Malafosse [13] has studied the spectrum and fine spectrum of the difference operator over the space s_r , where s_r denotes the Banach space of all sequences $x = (x_k)$ normed by

$$\|x\|_{s_r} = \sup_k \frac{|x_k|}{r^k}, \quad (r > 0).$$

The fine spectrum of the Zweier matrix operator Z^s over the sequence spaces l_1 and bv has been examined by Altay and Karakuş [7]. The fine spectrum of the generalized difference operator $B(r, s)$ over the sequence spaces c_0 and c has been determined by Altay and Başar [6]. Also, the fine spectrum of the operator $B(r, s)$ over the sequence spaces l_p and bv_p , where $1 < p < \infty$ has been determined by Bilgiç and Furkan [9].

Srivastava and Kumar [14] introduced the generalized difference operator Δ_v on the sequence space c_0 as follows: $\Delta_v : c_0 \rightarrow c_0$,

$$\Delta_v x = \Delta_v(x_k) = (v_k x_k - v_{k-1} x_{k-1})_{k=0}^{\infty} \text{ with } x_{-1} = v_{-1} = 0,$$

where $x = (x_k) \in c_0$ and (v_k) is either constant or strictly decreasing sequence of positive real numbers satisfying

$$\lim_{k \rightarrow \infty} v_k = L > 0 \text{ and } \sup_k v_k \leq 2L.$$

The fine spectrum of the operator Δ_v over the sequence spaces c_0 and l_1 has been determined by Srivastava and Kumar [14, 15]. Akhmedov and El-Shabrawy [4] have studied the spectrum of the operator Δ_v over the sequence space c . The same problem, in the case when the sequence (v_k) is assumed to be constant except for finitely many elements has been investigated by Akhmedov [3] over the sequence space l_p , where $1 \leq p < \infty$.

Our main focus in this paper is on the matrix Δ_v , where

$$\Delta_v = \begin{pmatrix} v_0 & 0 & 0 & \cdots \\ -v_0 & v_1 & 0 & \cdots \\ 0 & -v_1 & v_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Throughout this paper, we assume that (v_k) is a strictly decreasing sequence of positive real numbers satisfying $\lim_{k \rightarrow \infty} v_k = L > 0$ and $v_0 \leq 2L$. The case when (v_k) is constant sequence is not considered here.

The purpose of this paper is to study the spectrum of the operator Δ_v on the space l_p , where $1 < p < \infty$.

Now, we may give:

Lemma 1.1 [12, pp. 174]. Let $1 < p < \infty$ and let $A \in (l_\infty, l_\infty) \cap (l_1, l_1)$. Then $A \in (l_p, l_p)$.

Lemma 1.2 [10, pp. 59]. T has a dense range if and only if T^* is one to one.

2. The spectrum of the operator Δ_ν on the space l_p , ($1 < p < \infty$)

In this section, the spectrum, the point spectrum, the residual spectrum and the continuous spectrum of the operator Δ_ν on the space l_p , where $1 < p < \infty$ have been examined.

Theorem 2.1. $\Delta_\nu : l_p \rightarrow l_p$ is a bounded linear operator and $2^{\frac{1}{p}} \nu_0 \leq \|\Delta_\nu\|_{l_p} \leq 2\nu_0$.

Proof. The linearity of Δ_ν is trivial and so is omitted. Let us take any $x = (x_k) \in l_p$. Then, using Minkowski's inequality, we have

$$\begin{aligned} \|\Delta_\nu x\|_{l_p} &= \left(\sum_k |\nu_k x_k - \nu_{k-1} x_{k-1}|^p \right)^{\frac{1}{p}} \leq \left(\sum_k |\nu_k x_k|^p \right)^{\frac{1}{p}} + \left(\sum_k |\nu_{k-1} x_{k-1}|^p \right)^{\frac{1}{p}} \leq \\ &\leq \left(\sup_k |\nu_k|^p \right)^{\frac{1}{p}} \left(\sum_k |x_k|^p \right)^{\frac{1}{p}} + \left(\sup_k |\nu_{k-1}|^p \right)^{\frac{1}{p}} \left(\sum_k |x_{k-1}|^p \right)^{\frac{1}{p}} = 2\nu_0 \|x\|_{l_p}. \end{aligned}$$

Then $\|\Delta_\nu\|_{l_p} \leq 2\nu_0$.

On the other hand, let $y = (1, 0, 0, 0, \dots)$. Then

$$\|\Delta_\nu\|_{l_p} \geq \frac{\|\Delta_\nu y\|_{l_p}}{\|y\|_{l_p}} = \left(2|\nu_0|^p \right)^{\frac{1}{p}} = 2^{\frac{1}{p}} \nu_0.$$

Thus $\|\Delta_\nu\|_{l_p} \geq 2^{\frac{1}{p}} \nu_0$. This completes the proof. \square

Theorem 2.2. $\sigma(\Delta_\nu, l_p) = \{\lambda \in C : |\lambda - L| \leq L\}$.

Proof. First, we prove that $(\Delta_\nu - \lambda I)^{-1}$ exists and is in $B(l_p)$ for $\lambda \notin \{\lambda \in C : |\lambda - L| \leq L\}$ and then the operator $\Delta_\nu - \lambda I$ is not invertible for $\lambda \in \{\lambda \in C : |\lambda - L| \leq L\}$.

Let $\lambda \notin \{\lambda \in C : |\lambda - L| \leq L\}$. Then, $\lambda \neq \nu_k$ for all $k \in N$ and so $\Delta_\nu - \lambda I$ is triangle, and hence $(\Delta_\nu - \lambda I)^{-1}$ exists. Let $y = (y_k) \in l_p$ and solving the equation $(\Delta_\nu - \lambda I)x = y$, for $x = (x_k)$ in terms of y , we get

$$x_k = \frac{\nu_0 \nu_1 \dots \nu_{k-1}}{(\nu_0 - \lambda)(\nu_1 - \lambda) \dots (\nu_k - \lambda)} y_0 + \dots + \frac{\nu_{k-1}}{(\nu_{k-1} - \lambda)(\nu_k - \lambda)} y_{k-1} + \frac{1}{(\nu_k - \lambda)} y_k, \quad k \in N.$$

Then, $\sum_k |x_k| \leq \sum_k R_k |y_k|$, where

$$R_k = \frac{1}{|\nu_k - \lambda|} + \frac{\nu_k}{|\nu_k - \lambda| |\nu_{k+1} - \lambda|} + \frac{\nu_k \nu_{k+1}}{|\nu_k - \lambda| |\nu_{k+1} - \lambda| |\nu_{k+2} - \lambda|} + \dots, \quad k \in N.$$

Since $\lim_{k \rightarrow \infty} \left| \frac{v_k}{v_{k+1} - \lambda} \right| = \left| \frac{L}{L - \lambda} \right| < 1$. Then there exist $k_0 \in N$ and $q_0 < 1$ such that

$$\left| \frac{v_k}{v_{k+1} - \lambda} \right| < q_0, \text{ for all } k \geq k_0. \text{ Then, for each } k \geq k_0 + 1,$$

$$R_k \leq \frac{1}{|v_k - \lambda|} [1 + q_0 + q_0^2 + \dots].$$

But, there exist $k_1 \in N$ and a real number $q_1 < \frac{1}{L}$ such that $\frac{1}{|v_k - \lambda|} < q_1$, for all $k \geq k_1$. Then,

$$R_k \leq \frac{q_1}{1 - q_0},$$

for all $k > \max\{k_0, k_1\}$. Thus $\sup_k R_k < \infty$. This shows that $(\Delta_v - \lambda I)^{-1} \in (l_1, l_1)$.

Similarly, we can prove that $(\Delta_v - \lambda I)^{-1} \in (l_\infty, l_\infty)$, and so $(\Delta_v - \lambda I)^{-1} \in (l_1, l_1) \cap (l_\infty, l_\infty)$. Then, from Lemma 1.1, we have $(\Delta_v - \lambda I)^{-1} \in (l_p, l_p)$. Thus $\sigma(\Delta_v, l_p) \subseteq \{\lambda \in C : |\lambda - L| \leq L\}$.

Conversely, suppose that $\lambda \notin \sigma(\Delta_v, l_p)$. Then $(\Delta_v - \lambda I)^{-1} \in B(l_p)$. Since $(\Delta_v - \lambda I)^{-1}$ -transform of the unit sequence $e_1 = (1, 0, 0, \dots)$ is in l_p , we have

$$\lim_{k \rightarrow \infty} \left| \frac{v_k}{v_{k+1} - \lambda} \right|^p = \left| \frac{L}{L - \lambda} \right|^p \leq 1. \text{ Then } \{\lambda \in C : |\lambda - L| < L\} \subseteq \sigma(\Delta_v, l_p). \text{ But, } \sigma(\Delta_v, l_p)$$

is compact set, and so it is closed. Then $\{\lambda \in C : |\lambda - L| \leq L\} \subseteq \sigma(\Delta_v, l_p)$. This completes the proof. \square

Remark 2.1. From Theorem 2.1 and Theorem 2.2, we can easily prove that

$$2L \leq \|\Delta_v\|_{l_p} \leq 2v_0.$$

We should remark the reader from now on that the index p has different meanings in the notation of the spaces l_p , l_p^* and the point spectrums $\sigma_p(\Delta_v, l_p)$, $\sigma_p(\Delta_v^*, l_p^*)$ which occur in the following two theorems.

Theorem 2.3. $\sigma_p(\Delta_v, l_p) = \emptyset$.

Proof. Suppose $\Delta_v x = \lambda x$ for any x in l_p . Then

$$(v_0 - \lambda)x_0 = 0 \text{ and } -v_k x_k + (v_{k+1} - \lambda)x_{k+1} = 0, \text{ for all } k \in N.$$

Hence, for all $\lambda \notin \{v_k : k \in N\}$, we have $x_k = 0$, for all $k \in N$. So, $\lambda \notin \sigma_p(\Delta_v, l_p)$. This shows that $\sigma_p(\Delta_v, l_p) \subseteq \{v_k : k \in N\}$.

Now, let $\lambda = v_0$. If $x_0 = 0$, then $x_k = 0$, for all $k \geq 1$ and so $\lambda = v_0 \notin \sigma_p(\Delta_v, l_p)$. Also, if $x_0 \neq 0$, then $x_k = \frac{v_{k-1}}{v_k - v_0} x_{k-1} \neq 0$, for all $k \geq 1$ and

hence

$$|x_k| = \left| \frac{v_0 v_1 \dots v_{k-1}}{(v_1 - v_0)(v_2 - v_0) \dots (v_k - v_0)} \right| |x_0| > |x_0|,$$

for all $k \geq 1$. This contradicts the assumption that $x \in l_p$, and so $v_0 \notin \sigma_p(\Delta_v, l_p)$. Similarly, we can prove that $v_k \notin \sigma_p(\Delta_v, l_p)$, for all $k \geq 1$. Thus $\sigma_p(\Delta_v, l_p) = \emptyset$. \square

If $T : l_p \rightarrow l_p$, where $1 < p < \infty$, is a bounded linear operator with matrix A , then the adjoint operator $T^* : l_p^* \rightarrow l_p^*$ is defined by the transpose of the matrix A . It is well-known that the dual space l_p^* of l_p is isomorphic to l_q with $p^{-1} + q^{-1} = 1$.

Theorem 2.4. (i) $\{\lambda \in C : |\lambda - L| < L\} \cup \{v_0\} \subseteq \sigma_p(\Delta_v^*, l_p^*)$,

$$(ii) \left\{ \lambda \in C : \sup_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\} \subseteq \sigma_p(\Delta_v^*, l_p^*),$$

$$(iii) \sigma_p(\Delta_v^*, l_p^*) \subseteq \left\{ \lambda \in C : \inf_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\}.$$

Proof. Suppose that $\Delta_v^* f = \lambda f$ for $f = (f_0, f_1, f_2, \dots) \neq \theta$ in $l_p^* \cong l_q$, where $1 < p < \infty$ and $p^{-1} + q^{-1} = 1$. Then, by solving the system of equations

$$v_0 f_0 - v_0 f_1 = \lambda f_0, v_1 f_1 - v_0 f_2 = \lambda f_1, \dots, v_k f_k - v_k f_{k+1} = \lambda f_k, \dots$$

we obtain that $f_{k+1} = \frac{v_k - \lambda}{v_k} f_k$, $k \in N$. Then $f_0 \neq 0$, since $f \neq \theta$.

(i) It is clear that, for all $k \in N$, the vector $f = (f_0, f_1, \dots, f_k, 0, 0, \dots)$ is an eigenvector of the operator Δ_v^* corresponding to the eigenvalue $\lambda = v_k$, where $f_0 \neq 0$ and $f_n = \frac{v_{n-1} - \lambda}{v_{n-1}} f_{n-1}$, for all $1 \leq n \leq k$. Thus $\{v_k : k \in N\} \subseteq \sigma_p(\Delta_v^*, l_p^*)$.

Also, $\sum_k |f_k|^q < \infty$ if $\lim_{k \rightarrow \infty} \left| \frac{f_{k+1}}{f_k} \right|^q = \left| \frac{L - \lambda}{L} \right|^q < 1$. Thus

$$\{\lambda \in C : |\lambda - L| < L\} \cup \{v_0\} \subseteq \sigma_p(\Delta_v^*, l_p^*).$$

(ii) We have $f_k = \frac{(v_0 - \lambda)(v_1 - \lambda) \dots (v_{k-1} - \lambda)}{v_0 v_1 \dots v_{k-1}} f_0$, $k \geq 1$. Then

$$\sum_k |f_k|^q = |f_0|^q + \sum_{k=1}^{\infty} \left| \frac{(v_0 - \lambda)(v_1 - \lambda) \dots (v_{k-1} - \lambda)}{v_0 v_1 \dots v_{k-1}} \right|^q |f_0|^q \leq |f_0|^q + |f_0|^q \sum_{k=1}^{\infty} \left[\sup_n \left| \frac{v_n - \lambda}{v_n} \right| \right]^{kq}.$$

Thus, $\left\{ \lambda \in C : \sup_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\} \subseteq \sigma_p(\Delta_v^*, l_p^*)$.

(iii) Let $\lambda \in \sigma_p(\Delta_v^*, l_p^*)$. Then there exists $f \neq \theta$ in $l_p^* \cong l_q$ such that $\Delta_v^* f = \lambda f$.

Therefore the series $\sum_k |f_k|^q$ is convergent, and so,

$$\begin{aligned} |f_0|^q + |f_0|^q \sum_{k=1}^{\infty} \left[\inf_n \left| \frac{v_n - \lambda}{v_n} \right| \right]^{kq} &\leq |f_0|^q + \sum_{k=1}^{\infty} \left| \frac{(v_0 - \lambda)(v_1 - \lambda) \dots (v_{k-1} - \lambda)}{v_0 v_1 \dots v_{k-1}} \right|^q |f_0|^q = \\ &= \sum_k |f_k|^q < \infty. \end{aligned}$$

This implies that $\inf_n \left| \frac{v_n - \lambda}{v_n} \right| < 1$. \square

The equalities in Theorem 2.4 (i)-(iii) do not hold in general. Let us give the following example.

Example 2.1. Let $p = 2$ and consider the sequence (v_k) such that $v_k = \frac{k+3}{2k+5}$.

Clearly, (v_k) is a strictly decreasing sequence of positive real numbers and

$\lim_{k \rightarrow \infty} v_k = L = \frac{1}{2}$, $\sup_k v_k = v_0 = \frac{3}{5} \leq 1 = 2L$. For $\lambda = 1$, we have

$$\frac{\lambda - v_k}{v_k} = \frac{1 - v_k}{v_k} = \frac{k+2}{k+3}.$$

If we suppose that $\Delta_v^* f = (1)f$ for some $f = (f_0, f_1, f_2, \dots) \neq \theta$, then we obtain

that $f_0 \neq 0$, $f_k = \frac{v_{k-1} - 1}{v_{k-1}} f_{k-1}$, $k \geq 1$, and so

$$\sum_k |f_k|^2 = |f_0|^2 + |f_0|^2 \sum_{k=1}^{\infty} \left| \frac{1 - v_0}{v_0} \right|^2 \left| \frac{1 - v_1}{v_1} \right|^2 \dots \left| \frac{1 - v_{k-1}}{v_{k-1}} \right|^2 = |f_0|^2 + 4|f_0|^2 \sum_{k=1}^{\infty} \left(\frac{1}{k+2} \right)^2 < \infty.$$

Then $f \in l_2^* \cong l_2$ and so, $\lambda = 1 \in \sigma_p(\Delta_v^*, l_2^*)$. But $1 \notin \{ \lambda \in C : |\lambda - L| < L \} \cup \{ v_0 \}$ and

$$1 \notin \left\{ \lambda \in C : \sup_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\}.$$

On the other hand, if we consider the sequence (v_k) such that $v_k = k + 3 - \sqrt{k^2 + 5k + 6}$, then (v_k) is a strictly decreasing sequence of positive

real numbers and $\lim_{k \rightarrow \infty} v_k = L = \frac{1}{2}$, $\sup_k v_k = 3 - \sqrt{6} \leq 1 = 2L$. We can easily see that

$$1 \notin \sigma_p(\Delta_v^*, l_2^*) \text{ and } 1 \in \left\{ \lambda \in C : \inf_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\}.$$

Theorem 2.5. $\sigma_r(\Delta_v, l_p) = \sigma_p(\Delta_v^*, l_p^*)$.

Proof. The proof follows immediately from Theorem 2.3 and Lemma 1.2. \square

The following theorem follows immediately from Theorem 2.4 and Theorem 2.5.

Theorem 2.6. (i) $\{\lambda \in C : |\lambda - L| < L\} \cup \{v_0\} \subseteq \sigma_r(\Delta_v, l_p)$,

(ii) $\left\{ \lambda \in C : \sup_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\} \subseteq \sigma_r(\Delta_v, l_p)$,

(iii) $\sigma_r(\Delta_v, l_p) \subseteq \left\{ \lambda \in C : \inf_n \left| \frac{v_n - \lambda}{v_n} \right| < 1 \right\}$.

Theorem 2.7. $\sigma_c(\Delta_v, l_p) = \sigma(\Delta_v, l_p) \setminus \sigma_p(\Delta_v^*, l_p^*)$.

Proof. The proof follows immediately from Theorem 2.3 and Theorem 2.5. \square

The following theorem follows from Theorem 2.2, Theorem 2.4 and Theorem 2.7.

Theorem 2.8. (i) $\sigma_c(\Delta_v, l_p) \subseteq \{\lambda \in C : |\lambda - L| = L\} \setminus \{v_0\}$,

(ii) $\{\lambda \in C : |\lambda - L| \leq L\} \cap \left\{ \lambda \in C : \inf_n \left| \frac{v_n - \lambda}{v_n} \right| \geq 1 \right\} \subseteq \sigma_c(\Delta_v, l_p)$.

3. Conclusion

In this paper, some results concerning the spectrum, the point spectrum, the residual spectrum and the continuous spectrum of the operator Δ_v on the space l_p have been found. We have the following main results

$$\sigma(\Delta_v, l_p) = \{\lambda \in C : |\lambda - L| \leq L\},$$

$$\sigma_p(\Delta_v, l_p) = \emptyset.$$

In order to compute the residual spectrum and the continuous spectrum of the operator Δ_v over the space l_p in general, we first compute the series

$$S = \sum_k \left| \frac{(v_0 - \lambda)(v_1 - \lambda) \dots (v_k - \lambda)}{v_0 v_1 \dots v_k} \right|^q,$$

where $1 < p < \infty$ and $p^{-1} + q^{-1} = 1$. Then we have

$$\sigma_p(\Delta_v^*, l_p^*) = \{\lambda \in C : S \text{ is convergent series}\},$$

$$\sigma_r(\Delta_v, l_p) = \sigma_p(\Delta_v^*, l_p^*),$$

$$\sigma_c(\Delta_v, l_p) = \sigma(\Delta_v, l_p) \setminus \sigma_p(\Delta_v^*, l_p^*).$$

Acknowledgements

I wish to express my thanks to Prof. Ali M. Akhmedov, Baku State University, Faculty of Mechanics & Mathematics, for his kindly helps, careful readings and making useful comments on the earlier version of this paper which improved the presentation and its readability.

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l_p ($1 < p < \infty$) FƏZASINDA Δ_ν OPERATORUNUN SPEKTRİ HAQQINDA

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XÜLASƏ

İşin əsas məqsədi l_p ($1 < p < \infty$) fəzasında ümumiləşmiş Δ_ν fərq operatorunun spektrini müəyyən etməkdir. l_p ($1 < p < \infty$) fəzasında Δ_ν operatoru $\Delta_\nu x = \Delta_\nu (x_n) = (v_n x_n - v_{n-1} x_{n-1})_{n=0}^\infty$, $x_{-1} = v_{-1} = 0$ kimi təyin olunur. Burada $x = (x_n) \in l_p$ və (v_n) müəyyən şərtləri ödəyən ciddi azalan müsbət ədədlər ardıcılığıdır. Bu işdə Δ_ν operatorunun l_p fəzasında spektri, nöqtəvi spektri, qalıq spektri və kəsilməz spektri tapılmışdır.

Açar sözlər: operatorun spektri, ümumiləşmiş fərq operatoru, ardıcılıqlar fəzası.

О СПЕКТРЕ ОПЕРАТОРА Δ_v В ПРОСТРАНСТВЕ l_p ($1 < p < \infty$)

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РЕЗЮМЕ

Основной целью настоящей работы является нахождение спектра обобщенного разностного оператора Δ_v в пространстве l_p ($1 < p < \infty$). Оператор Δ_v в пространстве l_p определяется следующим образом: $\Delta_v x = \Delta_v (x_n) = (v_n x_n - v_{n-1} x_{n-1})_{n=0}^{\infty}$, $x_{-1} = v_{-1} = 0$. Здесь $x = (x_n) \in l_p$ и (v_n) строго убывающая последовательность действительных чисел, удовлетворяющая некоторым дополнительным условиям. В этой работе были найдены спектр, точечный спектр, остаточный спектр, непрерывный спектр оператора Δ_v в пространстве l_p .

Ключевые слова: спектр оператора, обобщенный разностный оператор, пространство последовательностей.

Поступила в редакцию: 31.01.2011 г.

Принято к печати: 03.10.2011 г.